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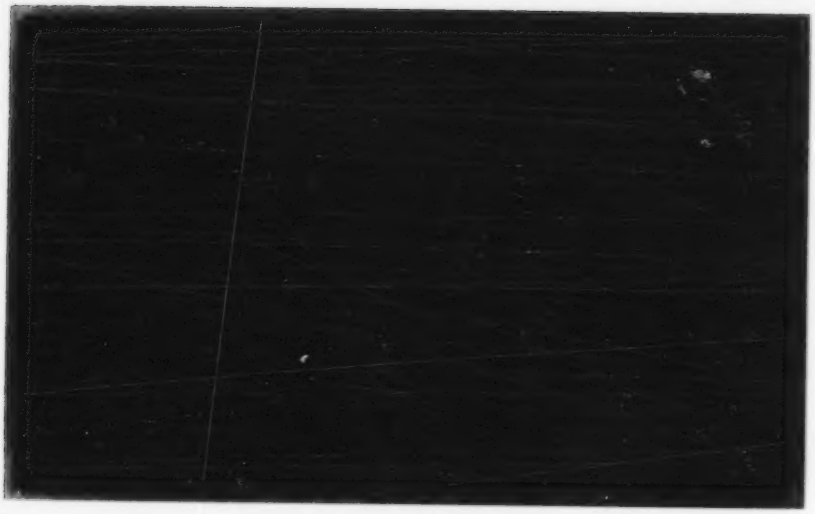

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Long-Range Planning of Power Distribution Systems : Primary Networks

by

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ABSTRACT

This paper focuses on the expansion planning of power distribution systems. An optimization model that considers substation location and sizing, primary feeder routing and reconductoring simultaneously is used to design expansion plans. The model considers the fixed and variable costs for all new elements and also for the existing ones when expansion is taken into account.

The objective function seeks to minimize the total annualized expansion cost that includes the cost of losses modeled as a quadratic function. An efficient implicit enumeration algorithm combined with several heuristic procedures allows to solve large expansion problems in very reasonable time using microcomputers. The radiality is obtained by applying heuristics that deal with the true expansion costs. The voltage drop constraints are checked in the final radial solution and corrective actions are performed if they are not satisfied.

Keywords: Power distribution planning, capacited location problem, capacity expansion models.

RÉSUMÉ

On décrit dans ce travail un modèle d'optimisation pour la planification de l'expansion des systèmes de distribution d'énergie électrique. On considère simultanément la localisation et le dimensionnement des postes, la localisation des routes pour les conducteurs et aussi la possibilité de modifier les câbles déjà existants dans le système. Le modèle prend compte des coûts fixes et variables pour les équipements nouveaux, comme pour les équipements déjà existants dans un réseau.

La fonction objective, qu'on cherche à minimiser, est le coût total actualisé, comprenant le coût des pertes représenté comme une fonction quadratique. On utilise un algorithme d'énumération implicite combiné avec des procédures heuristiques, ce qui permet de résoudre, par micro-ordinateur, des problèmes d'expansion de grande taille avec des temps raisonnables. Les contraintes de radialité du réseau sont satisfaites après l'application d'une heuristique qui tient compte des coûts réels d'expansion. Les contraintes de chute de tension sont vérifiées pour la solution radiale finale, et si elles ne sont pas satisfaites, des actions correctives sont prises.

Mots-clés: planification de la distribution d'énergie électrique, modèles de localisation, modèles d'expansion de capacité.

1. Introduction

The planning of substation and primary feeder facilities in electric power distribution systems is a subject that has attracted great attention in the last two decades [1–11]. Part of this interest is related to some special features of distribution systems. The costs of building, operating and maintenance are very large which encourage the search for better expansion alternatives. Moreover, the major part of the global losses occurs in distribution systems. Hence, improvements in network design represent valuable savings.

Planning the expansion of distribution networks is a problem faced by utilities that experience demand growth. The concept of horizon year is used to design a network that meets the consumers' horizon year demand. In this static approach the planning is concerned with the optimal location and sizing of new substations as well as with the expansion of the existing ones. Another problem is to determine the primary feeder routes, the best conductor size, the reconductoring and, in most cases, a radial topology of the feeder system. The optimization criterion is basically economic and it attempts to minimize the sum of the total investment cost and the cost of losses. A feasible solution has to satisfy the consumers' demand, the Kirchhoff's current law, the power capacity limits, the voltage drop and radiality constraints. This problem presents a formidable combinatorial nature that make its global solution quite difficult. In spite of this, many authors have proposed mathematical models and solution strategies to address not all but part of the planning aspects. It is worth noting that substation location decisions are interrelated with feeder route design.

Hence, better results can be expected from models that consider simultaneously these two planning aspects [3–9].

Most of these models are mixed-integer linear programming formulations where the fixed costs of substations and feeders are represented by binary variables. The feeder network has been modeled either as a transportation network [3–6] or as a network flow model [7–9]. The latter is more realistic since it is able to represent each feeder section independently. More recently expert systems have been developed for distribution system planning [15].

It should be pointed out that the network expansion problem is a dynamic decision problem where the planner faces a continously changing demand pattern. Two approaches have been proposed to consider the time aspect of planning. Sun et al. [6] suggested a pseudo-dynamic approach where, at first, a static optimal plan is obtained for the horizon year. Afterwards, the static method is applied to each intermediate year selecting equipment from the set that has been specified for the horizon year static optimum system. One cannot guarantee that the final solution is optimal but this methodology has been widely applied for large scale distribution network planning. The completely-dynamic approach [10] considers a dynamic mixed-integer programming model to solve the optimal sizing, timing, and location of substation and feeder expansion problem simultaneously. Although this method yields an optimal solution, the authors have not reported applications to practical problems. Youssef and Hackan [11] also propose a dynamic approach and compare the solutions obtained by static and dynamic methods.

In this article we propose a new static methodology for distribution system expansion planning which makes use of a nonlinear mixed-integer programming model and is able to deal with large networks. The substation location problem and the feeder routing and sizing problem are consider simultaneously as well as the decisions concerning the reconductoring of feeder sections. The cost associated with demand and energy losses is represented as a quadratic function and the substation and feeder fixed costs are modeled as binary variables. The feeder network is modeled as a network flow problem where each feeder section can be represented independently. The quadratic objective function being a realistic description of the loss aspect, yields solutions where the feeder power flows present a balanced pattern. A linear representation of the power losses has been employed by most methods because linear models are easier to solve and the solutions obtained are, in general, radial. On the other hand, the pattern of the feeder power flows are often unsatisfactory due to the flow discrepancy between feeders. In our method the radiality and voltage drop constraints are not considered explicitly in the model since they introduce additional binary variables which would prevent its application to large networks. Instead, we suggest heuristic procedures to ensure these operating aspects of distribution networks.

2. Mathematical Modeling

An existing distribution system can be suitably represented by a non-oriented graph G(N,M) where N is the node set and M is the edge set. The node set is constituted by supply nodes (substations) and demand nodes which are defined on the basis of small geographical areas. Each area, which can comprise one or more

transformers, has assigned a demand load. An edge connects two nodes and represents an existing primary feeder section. Each edge has associated with it the type and size of the existing feeder cable as well as the length of the feeder section.

It is possible that the studies carried out to forecast the horizon year demand determine the necessity to create new demand nodes. These nodes must be connected to the existing network through new edges which constitutes an extended graph that should be considered for planning studies. Each substation is connected by a special arc to a node, called energy source node, which represents the fictitious source of power consumed by the demand nodes.

Our model considers two kinds of costs: investment cost and cost of losses. Investment cost comprises building and equipment costs and the cost of losses includes the cost of energy and demand losses. Usually, losses are calculated on annual basis and correspond to the value of the energy and the lost instaled capacity due to the Joule effect on conductors.Both costs of losses are quadratic functions of peak power flow. It is convenient that the total cost function be presented as an annual equivalent cost in order to take into account the temporal aspect of the problem. Indeed, investment costs occur instantaneously, where as the losses span over the whole equipment life.

3. Feeder Cost

In order to model the feeder cost function we must consider whether the feeder section is a new one or an existing one. For a new feeder section the model has to provide answers to two basic questions: if the feeder section should be constructed or not and, in the affirmative case, what is the conductor size. Figure 1 shows the

annualized total cost functions for a set of conductors normally used in distribution networks. Note that depending on the feeder's power flow there is an economical feeder size.

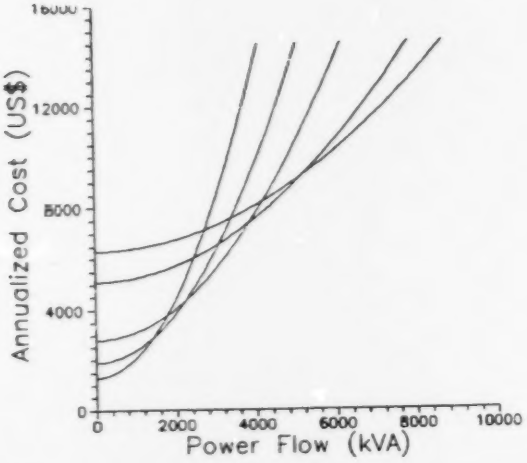


Figure 1 : Annualized feeder cost functions

Consider now the best inferior linear approximation to the curves. This linear function introduces a fictitious fixed cost which can be modeled by a binary variable, which takes the value one if the feeder section is to be built and zero, otherwise. Thus, the linear approximation is able to answer both questions since the optimal power flow determined by the optimization method defines the optimal feeder size.

For an existing feeder section one must take into account that depending on the horizon year's feeder flow the existing conductor should be exchanged by a more economical one. We will refer this operation as reconductoring. Figure 2 shows the cost function of an existing feeder section and the linear approximation function which represents building a new feeder section. Note that beyond the crossing point it is advantageous to replace the existing conductor by a larger one. So the

resulting continuous curve is able to represent the reconductoring cost function. It is worth noting that the graph that results from these feeder cost representations is uncapacitated. This important fact allows our model to respect Kirchhoff's second law in a very simple way. In addition, the feeder cost functions adopted allow the use of just one edge to model each feeder section, leading to a compact network representation.

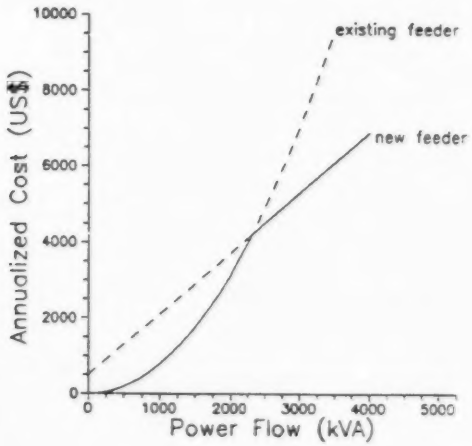


Figure 2 : Reconductoring cost function

4. Substation Cost

In the distribution system expansion problem there are two decisions concerning substations: which new substations should be built, if any, and which existing substations deserve to be expanded. A node that represents a substation is connected to the energy source node by one or more arcs. If the node represents a candidate substation, the corresponding cost function must take into account the fixed cost and the cost of losses. Let us suppose that, at an specific candidate node, equipment are available for installation at 10MVA or 25MVA. We must create two parallel arcs —

each one associated to the respective cost function — connected to the source node and two binary variables to represent the fixed cost associated to each substation size. The cost functions and its graph representations are shown in Figure 3a and 3b, respectively.

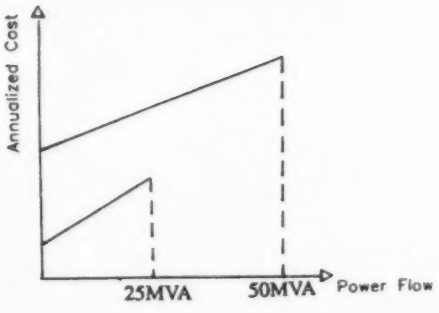


Figure 3a : New substation cost functions

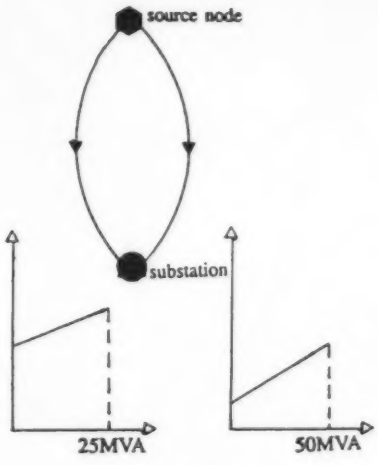


Figure 3b : Graph representations

It should be noticed that the ratio between the fixed and variable costs for a substation is much higher than the corresponding ratio for a feeder. Hence, a linear variable cost function can be adopted for substations. On the other hand, the linear approximation used to model new feeder cost is not suitable in this case since it introduces rough errors. The decision on which substation will be installed, if any, is taken during the resolution process.

If the node corresponds to an existing substation, we are dealing with an expansion decision. The existing power supply capacity is represented by an arc connected to the energy source node without a binary variable associated to it. Expansion alternatives are taken into account in the same way as the new substation modelling. The complete

mathematical model can be tought as a capacitated quadratic location problem [12] and can be stated as:

$$\text{Minimize} \qquad \sum_{j \epsilon M} c_j \, x_j^s + F_j y_j \tag{1}$$

$$\text{Subject to} \qquad Ax = b \tag{2}$$

$$x \leq y\overline{x} \tag{3}$$

$$y \epsilon \{0, 1\} \tag{4}$$

$$\text{voltage drop constraints} \tag{5}$$

$$\text{radiality constraints} \tag{6}$$

where

M	set of edges (feeder sections) and arcs (substations)
A	Node-edge incidence matrix
x	Power flow vector
b	Power demand vector
c_j	Annualized total variable cost for element j
F_j	Annualized fixed cost for element j
y	Binary variable vector
$\overline{x}$	Power flow upper bound vector
s $\begin{cases} 1 \\ 2 \end{cases}$	if j represents a new substation or a new feeder section if j represents an existing feeder section

The objective function seeks to minimize the annual variable cost plus the annualized fixed cost. For existing elements, the fixed cost F_j equals zero and the associated binary decision variable is set to unity. The set of constraints (2) guarantee that each demand is met, at the same time the Kirchhoff's current law is respected. Constraints (3) ensure that there is no power flow in an element unless it actually exists (i.e. $y_j = 1$). They also guarantee that the power flow does not exceed the maximum capacity of the element. The solution integrality is assured by constraints (4). The formulation (1) — (4) constitutes the classical capacitated location problem for which we have developed an efficient solution technique. The explicit consideration of voltage drop and radiality constrainsts will generate a large number of additional binary variables which would restrict the method to solve small networks. Moreover, it would not be possible to exploit the network flow structure and the efficient algorithms specially designed for solving network problems. The important aspects of voltage drop and radiality will be addressed later by appropriate procedures.

5. Choosing Candidate Sites for Substations

Large distribution expansion problems combined with high demand growth rates may impose a difficult question to planners prior to select the new substation locations: how to choose the substation candidate node set that will be used in the model (1) — (4) ? In general, this set is chosen by the planner based on his experience. In order to help planners to choose good initial sites for substation location we propose another computational tool. It is based on a mathematical model that attempts to

find the load centers of the distribution network. Specifically, the model looks for a fixed number of nodes-say p-among all network nodes, that minimizes the resistence x kVA product. Consider that each existing feeder section has a known resistence value. Compute the paths of minimal resistence between each pair of nodes. This can be accomplished by any shortest path algorithm [12]. Consider now a pair of nodes (k, j) and its associated minimal resistence value r_{kj}. Let $d_{kj} = r_{kj} b_k$ be the minimal resistence x kVA product where b_k is the load associated to node k. It can be viewed as the minimal electrical momentum to assign node k to node j. Finding p nodes that minimize the total momentum of the network corresponds to locating the p load centers of the network. These nodes are known as the graph medians.

This problem can be formulated as a uncapacitated location problem, also called the p-median problem:

$$Minimize \qquad \sum_{k \epsilon N} \sum_{j \epsilon N} d_{kj}\, x_{kj} \tag{7}$$

$$\text{Subject to} \qquad \sum_{j \epsilon N} x_{kj} = 1 \qquad k \epsilon N \tag{8}$$

$$\sum_{j \epsilon N} x_{jj} = p \tag{9}$$

$$x_{kj} \leq x_{jj} \qquad k, j \epsilon N \tag{10}$$

$$x_{kj} \epsilon \{0, 1\} \qquad k, j \epsilon N \tag{11}$$

where

N		node set of the graph
p		number of medians
d_{kj}		minimal resistance x kVA product between nodes k and j
x_{kj}	$\begin{cases} 1 \\ 0 \end{cases}$	if node k is assigned to median j otherwise

The objective function seeks to minimize the total electrical momentum. Constraints (8) ensure that each node is assigned to only one median. Constraint (9) limits the number of medians to p. The constraint set (10) ensures that each node is only assigned to median nodes and (11) guarantee the solution integrality. Notice that the x_{jj} variables corresponding to existing substations must be set to unity. On the other hand, the planner can forbid some nodes to be medians. We believe that an appropriate choice for p is $3\left\lceil \frac{\Delta b}{\overline{C}} \right\rceil$,

where

Δb	power demand that exceeds the total existing installed capacity
$\overline{C}$	smallest substation size
$\lceil . \rceil$	means the greatest integer less or equal to

Remember that the p medians are only candidate nodes. The decision about substation locations is left to the capacitated model which selects an optimal solution from the p set. Therefore, the number p has to be chosen in such a way that it does not restrict the search process.

6. Solution Techniques

In this section we present optimization procedures for solving the p-median problem and the capacitated location problem, as well as some heuristics developed to generate improving radial solutions.

p-median problem

There is a great variety of solution techniques proposed for solving the p-median problem which ranges from simple heuristics [13] to optimal algorithms [19]. For large scale problems (hundreds of nodes), the optimal algorithms are generally found to be too expensive in terms of memory space and computation time. We have implemented a "greedy" type heuristic which has shown to be very efficient and effective for solving the p-median problem [13].

First, a matrix of minimal resistence between all pair of nodes is calculated. For the sake of simplicity we adapt a minimum cost network flow algorithm (MCNF) for this purpose. The MCNF algorithm [12] is the same used intensively in the solution technique applied for solving the substation location and feeder routing problem. But any shortest path algorithm can be used [12]. The greedy heuristic first chooses a node that solves (7)-(11) for $p = 1$. Then it proceeds recursively until it selects all the p medians. At an intermediate iteration, the heuristic finds the unselected node that yields the greatest improvement in the value of the objective function.

For better results an interchange heuristic [14] can be applied after the greedy solution have been found. It attempts to improve the greedy solution by interchanging

a node currently in the solution with one that is not in. This procedure continues until no improvement results from the interchanges.

Substation Location and Feeder Routing

Problem (1)-(4) is solved by a branch and bound method which determines the optimal horizon year configuration. The method employs a classical implicit enumeration scheme. This article only deals with the main manipulations necessary to implement the method. A more detailed description can be found in [12] or [18].

1. A lower bound to the optimal solution of (1)-(4) is obtained by solving its linear relaxed version. This can be accomplished by relaxing constraints (4), which allows the binary variables to assume values in the [0,1] interval.
2. At each node of the binary tree a relaxed problem is solved by a MCNF algorithm [12]. In a feeder section the power flow and its associated cost function are modeled by only one arc, assuming that in one direction the flow is positive and in the other one it is negative. This compact representation permits savings in storage space and it improves the running time.
3. The nonlinear part of the feeder cost functions is approximated by a piecewise linear function in order to exploit the efficiency of the MCNF's linear structure. The use of pointers enables us to represent the piecewise linearization through only one arc.
4. Basis change operations performed by the MCNF algorithm are greatly improved by using a special technique, called multiple iteration [17], that allows a sequence

of basis changes without modifying the structure of the tree. The average time savings observed were about 30%.

5. The existing feeder cost function is concave because it includes the reconductoring possibility. To account for these concavities a heuristic procedure is employed. Figure 4a shows a piecewise linear approximation of a feeder section's reconductoring curve. Suppose that the solution of a relaxed problem, where all reconductoring curves are approximated by piecewise linear functions, gives x^* as the feeder flow value. The piecewise linear cost functions are adjusted, as shown in Figure 4b, and a new solution is obtained using the MCNF routine. This iterative procudure is repeated until the flow values remain unchangeable. This heuristic has been tested on several examples and it has always presented a convergent behavior in few iterations.

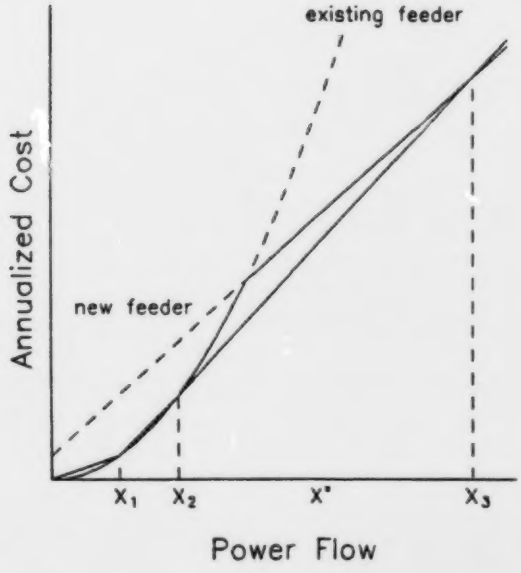


Figure 4a : Piecewise linear approximation

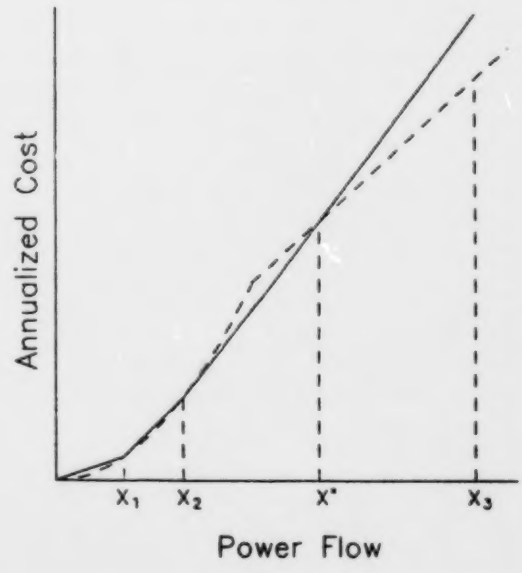


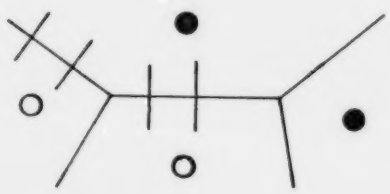
Figure 4b : New cost function

Radial Solution

The optimal solution obtained by the branch and bound search may not be radial since the objective function ia a nonlinear convex function. It is possible then that there exist cycles in the network, which means that there are demand nodes served by two different feeder routes. A radial configuration can be accomplished by a heuristic algorithm that, in each cycle of the network, looks for the arc with the lowest flow value. The flow value of the arcs is set to zero by forcing a flow movement in the cycle. At the end of this procedure a radial network results. The underlying idea of this heuristic is to yield a radial solution as close as possible to the nonlinear solution, that represents the less expensive expansion plan.

Feasibility Procedure

If any substation become infeasible by the previous procedure, a new heuristic algorithm seeks to transfer load from overloaded substations to other ones with idle capacity. First, we identify all the arcs connecting the overloaded substations to feasible substations. These arcs are depicted in the example shown in Figure 5.



● **overloaded substations**
○ **feasible substations**

Figure 5 : Candidate arcs for load transfers

In the next step we select the arc that minimizes the difference between the true cost to meet the load assigned to the connecting arcs and the current cost. The procedure is repeated until all substations become feasible. If the neighbours that surround an overloaded substation are infeasible too, the capacities of the neighbouring substations are increased by 1% and the exchange process is repeated.

Feeder Load Transfer

The current feasible radial solution can be substantially improved by a simple analysis of the feeder's flow profile. Consider all the feeders which emanate from a substation and identify the arcs that connect the feeders but do not belong to the radial solution. These arcs correspond to existing or designed arcs for which the previous optimization procedures have assigned a zero flow. Now we attempt to insert them in the current radial solution, in such a way that load is transfered from an overloaded feeder to another one with idle capacity. A heuristic procedure tests all possible exchanges. An exchange is performed if it is considered advantageous in terms of a true cost evaluation.

Voltage Drop

It is well known that distribution planning optimization models that consider nonlinear cost of losses in the objective function penalize excessive voltage drop. Hence, it is expected that few nodes, if any, will exceed the allowable limit. In this case, a routine identify the subtree that contains the infeasible node. If in the path connecting this node to its corresponding substation there is any new feeder section, the sizes of the conductors in this section are gradually increased. If the

node remains infeasible, we try to transfer the subtree to other feeder or to install a corrective equipment. Both costs are evaluated and the less expensive alternative is chosen.

7. Computational Application

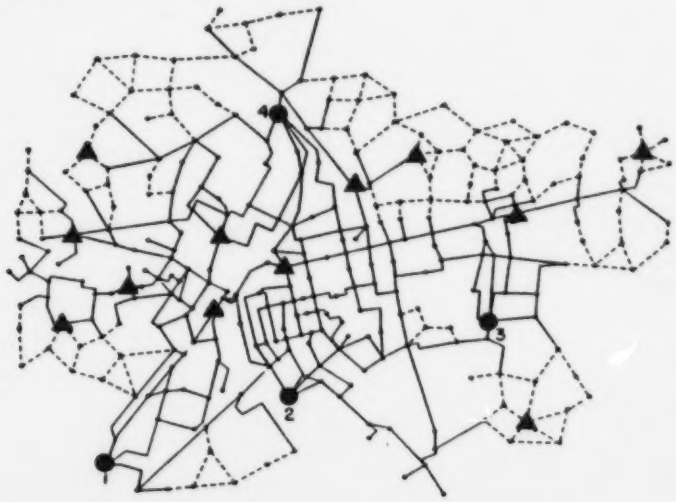
The optimization algorithms presented in the last section has been implemented in C language and all computational tests are performed in a 286 microcomputer with an arithmetic coprocessor.

The whole methodology has been tested in a real distribution network that serves a 300 000 people brazilian city, characterized by a rapid demand growth. First, the optimization process is used to achieve a static solution to meet the required demand in the year 2000. Afterwards, the single-stage model is successively applied to 1993 and 1996, choosing elements from the set of equipment that belongs to the 2000 year's plan. This time-phased planning follows the pseudo dynamic approach suggested by Sun et al. [6]. Due to space limit, we only report in this paper the results for the horizon year planning. The complete time-phased planning can be found in [15].

Candidate Nodes Selection

The greedy algorithm is used to find the medians of the distribution network. These medians represent the candidate nodes for installing new substations. Figure 6 shows the graph that represents the primary network. The dashed arcs represent possible feeder routes for serving future load points. There are 155 dashed arcs. The complete graph has 301 nodes and 438 arcs. There are four 25MVA substations in

1990 which is considered the base year of the planning period. The solution found by the greedy algorithm has 12 medians.



● existing substations

▲ new substation candidate nodes (medians)

Figure 6 : Candidate nodes for siting new substations

Substation Location and Feeder Routing

Three of the four existing substations can be expanded, (substations 1, 2 and 3) which means that the location model (1)-(4) that solves the substation location and feeder routing problem has 170 binary variables (15 substation candidate nodes plus 155 feeder section candidate arcs). It is assumed that substations are available in a standard rating of 25MVA and conductors in five different sizes.

The branch and bound algorithm is applied to find the new substation and feeder layout. The fathoming criterion is based on a tolerance of 5%. The horizon year solution network is presented in Figure 7. Three new 25MVA substations are located (5, 6 and 7) and substation 2 is expanded to 50MVA.

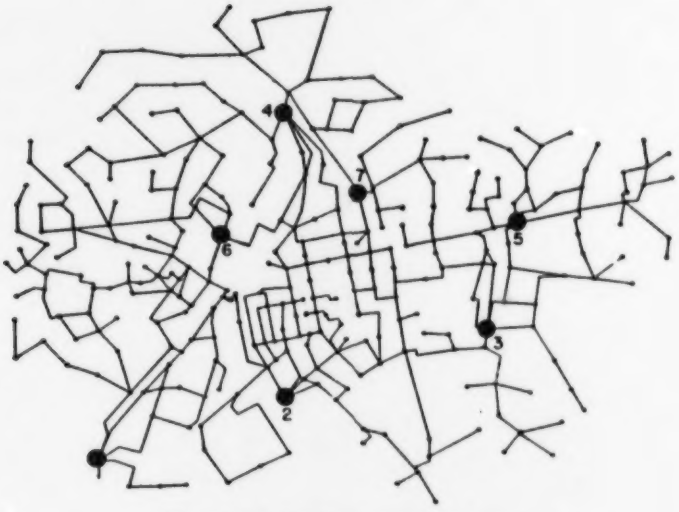


Figure 7 : Horizon year target system

It should be noticed that the optimal solution is a meshed network due to the nonlinear objective function. The main feature of this solution is its well balanced power flow profile.

Heuristics Procedures

The solution obtained by the heuristic procedure that transforms the meshed network into a radial system is depicted in Figure 8.

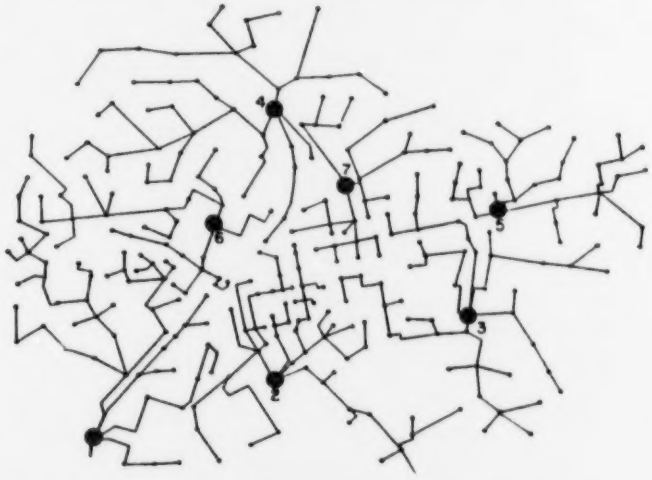


Figure 8 : Horizon year radial system

The substation load sharings (MVA) are:

1	2	3	4	5	6	7
25.1	44.3	22.8	25.1	26.7	23.3	24.9

Substations 1, 4 and 5 have their capacity exceeded in the radial solution thus determining the use of the feasibility procedure. A radial feasible solution is obtained by exchanging only four load points located on the serving area borders. The new substation loads become:

1	2	3	4	5	6	7
24.7	44.3	24.5	24.6	25.0	24.2	24.9

Afterwards, the feeder load transfer algorithm is applied to improve the radial feasible solution. Figure 9 shows the balanced network where the new arcs introduced into the solution by both heuristic procedures are depicted in bold and the arcs which are deleted are identified by the dotted segments. That is considered the horizon year final distribution configuration. The voltage drop calculations are carried out for the nodes of the final system and none of them was found beyond the specified limit (5%).

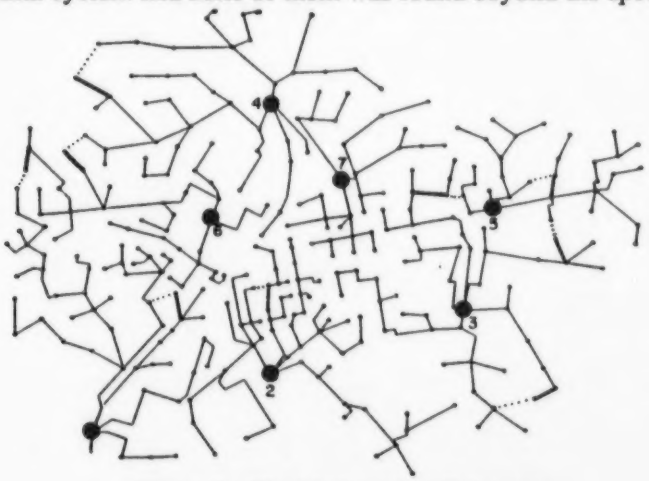


Figure 9 : Final balanced radial system

Table 1 shows the objective function value and the running times for each problem solved in the proposed methodology. The objective function value represents the equivalent annual cost and is expressed in 10 US$.

Table 1 : Computational Results

Problem	Objective Function Value	Variation* (%)	Running+ Time (sec.)
p-median	-	-	2
Branch and Bound	1.322	-	580
Radial Solution	1.382	(+) 4.5	0.94
Feasible Solution	1.385	(+) 0.2	0.22
Balanced Solution	1.355	(-) 2.1	0.66

*(-) improvement + 286 microcomputer
(+) disimprovement

Table 2 shows, in percentages, the parts that constitute the cost of the final solution.

Table 2 : Final Cost Composition

New substation construction	42.7
Existing substation expansion	11.2
Feeder construction	7.5
Losses	38.6

It is interesting to compare the solution achieved by the proposed methodology with that obtained by the linear approach, which considers the costs associated with

all arcs as linear functions. The radial solution obtained by the linear approach is 8% more expensive.

8. Conclusions

This article proposes a new methodology for solving the distribution network expansion problem which is formulated as a nonlinear mixed-integer programming model. The main relevant aspects concerning static distribution planning are included in the mathematical model which considers the fixed variable costs for substations and feeders, the cost of losses and the reconductoring cost. The static methodology uses an efficient branch and bound algorithm to select a meshed horizon year distribution configuration which is tranformed into an improved radial system by several heuristic procedures. The most important feature of a solution obtained by the methodology is the well balanced power flow pattern which yields a low cost expansion plan. Furthermore, we believe that this also explains the fact that all demand points have their voltage drop constraints satisfied.

The use of efficient optimization techniques allows solving instances of up to 1000 nodes in microcomputers. Therefore, we believe that this approach is useful to handle detailed analysis of distribution expansion problems which should result in more accurate plans.

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